



TONBRIDGE SCHOOL

Scholarship Examination 2008

MATHEMATICS I

**Tuesday 29th April 2008
9.00 a.m.**

Time allowed: 1 hour 30 minutes

*Answer as many questions as you can.
Questions 1 to 5 are worth 8 marks each;
Questions 6 to 9 are worth 15 marks each.*

*All answers must be supported by adequate explanation.
Calculators may be used in any question.*

1. Solve the simultaneous equations $5x + 2y = 11$ and $7x + 5y = 11$ and give a check that your answers are correct.

8/8 [8 marks] ✓

2. The rear wheels of a tractor have radii 0.65 m. The tractor makes a journey of 2 km.

(a) How many revolutions do the rear wheels make during this journey? ✓

(b) If the smaller front wheels make 1450 revolutions on the same journey, what is their radii? Division wrong



6/8 [8]

3. A sequence of numbers is constructed in the following way.

The **FIRST NUMBER** = 3

and **NEXT NUMBER** = $\frac{\text{TWICE PRESENT NUMBER} - 1}{\text{TWICE PRESENT NUMBER}}$. (3)

- (a) Work out the first 5 terms of the sequence, leaving your answers as fractions. What do you notice? ✓
- (b) Use your answer to (a) to predict both the 27th term and the sum of the first 27 terms of the sequence.

[8]

4. In appropriate units, the radius R of a star is related to its brightness L and its surface temperature T by the formula $R = \frac{12\sqrt{L}}{T^2}$.

(a) If $L = 8.2$ and $T = 5.7$, find the value of R .

(b) If $L = 3.8$ and $R = 0.9$, find the value of T . ✓ Done

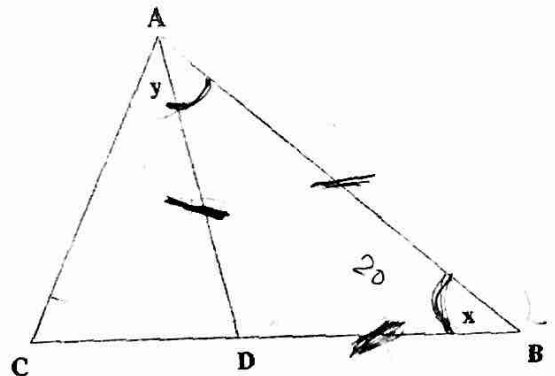
(c) If $R = 3.4$ and $T = 2.5$, find the value of L .

8/8

[8]

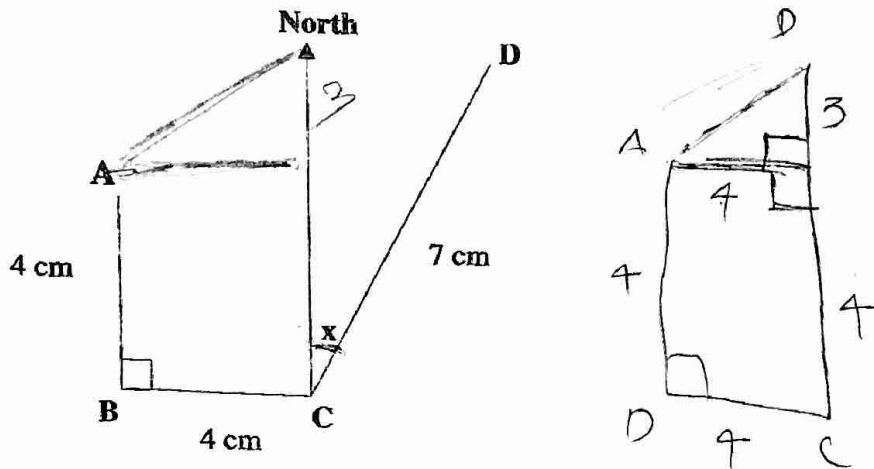
5. In the figure, $AB = BC$ (so triangle ABC is isosceles) and $AC = AD$ (so triangle ACD is isosceles). Also, angle $ABC = x$ and angle $CAD = y$.

- (a) If $x = 20^\circ$, find y . ✓
- (b) In general, use algebra to express y in terms of x . $y = x$
- (c) If triangle ABD is also isosceles, find x .



[8]

6. The diagram shows three joined rods of lengths 4 cm, 4 cm and 7 cm lying on a flat surface. Rods AB and BC are fixed, but rod CD is able to rotate around its end C. Point A is North of B and the bearing of D from C is x degrees.



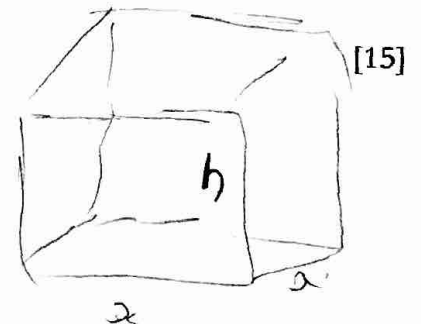
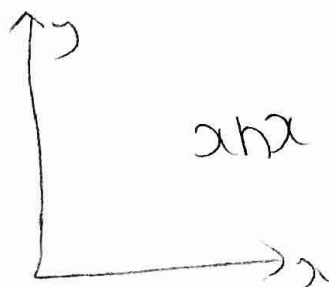
- (a) Use Pythagoras's Theorem to find the distance AD in each of the following cases:
 (i) $x = 0^\circ$, (ii) $x = 90^\circ$, (iii) $x = 180^\circ$, (iv) $x = 270^\circ$.
- (b) If CD does a full rotation through 360° , what are the largest and smallest possible values of the distance AD and at which values of x do they occur?



[15]

7. A solid cuboid has a square base with side-length x cm. The total volume of the cuboid is fixed at 20 cm^3 . The total surface area, $y \text{ cm}^2$, is given by the formula $y = 2x^2 + \frac{80}{x}$.

- (a) Calculate the values of y corresponding to $x = 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5$.
- (b) Choosing suitable scales, plot a graph of y against x .
- (c) Which value of x gives the smallest value of y ? $x = 2.5$
 What is the corresponding height of the cuboid?
- (d) Why is it hard to be sure of your answer to (c)?
 How would you obtain a more accurate answer?



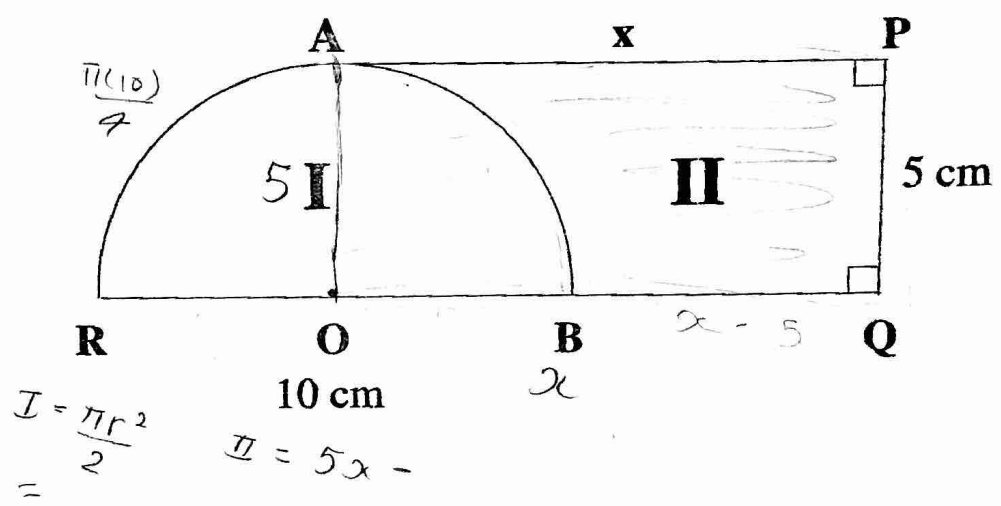
$$V = x^2 h = 20$$

$$\frac{2.5^2 h}{2.5^2} = \frac{20}{2.5^2}$$

[15]

8. The figure below shows a semicircle with centre O and diameter RB = 10 cm which overlaps a rectangle APQO where A is at the top of the circle, RBQ is a straight line and PQ = 5 cm, the radius of the semicircle. The side AP of the rectangle APQO has length x cm.

- (a) Find x if the area of the semicircle (labelled I) is equal to the area of the region labelled II.
- (b) Find x if the distance from A to B around the perimeter of the figure via P and Q is equal to the distance from A to B around the perimeter of the figure via R and O.



[15]

9. In the pyramid below, each box contains a number which is the sum of the two numbers in the boxes underneath it on which it rests. Some numbers have already been filled in. Suppose that the top-most number is x.

- (a) Make a rough copy of the pyramid and fill in the empty boxes in terms of x. (To get you started, the box to the left of the box containing the number 45 should be filled in with the expression 'x - 45'.)
- (b) Find x, given that the bottom row in fact consists of five different single-digit whole numbers.

